Álgebra Linear Numérica

PageRank: Uma Aplicação de Algebra Linear

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Faça muitas perguntas
Somos todos humanos

Plano de Aula

Vimos

- Matrizes e Grafos
- Matriz de Adjacência tem importância profunda
- Normalizando matrizes de adjacência *W*:
  - Matrizes estocásticas *P*
  - Grafo vira cadeia de Markov
- Multiplicação de matrizes permite difusão no grafo

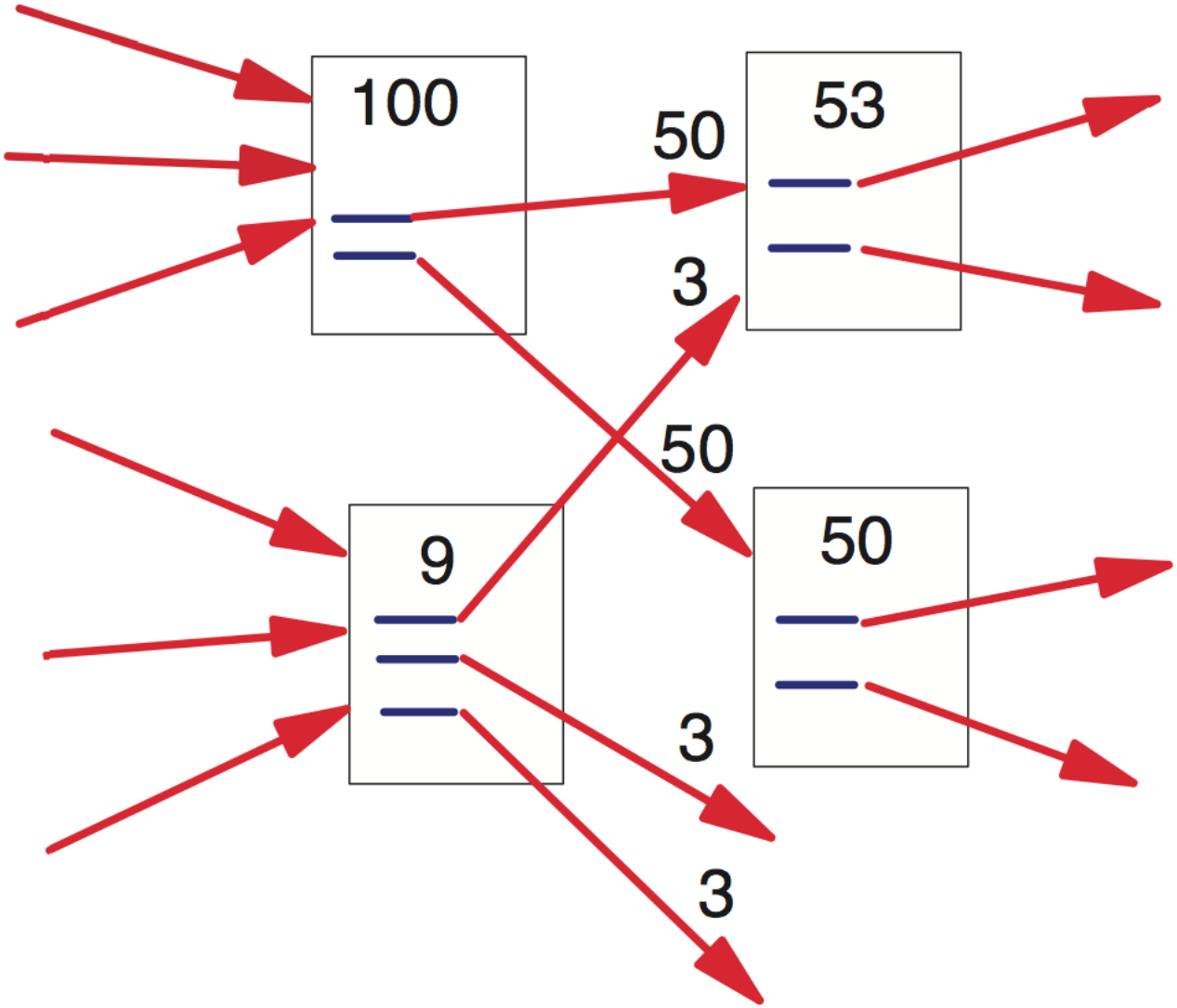
Plano de Aula

Hoje, 3/Julho/2013

- Grafos e o conceito de PageRank
- Grafos como Cadeias de Markov
- Matrizes de Adjacência Estocásticas
- PageRank como limite de um processo de Markov
- PageRank como autovetor da matriz de Markov
- Método das potências para autovetores

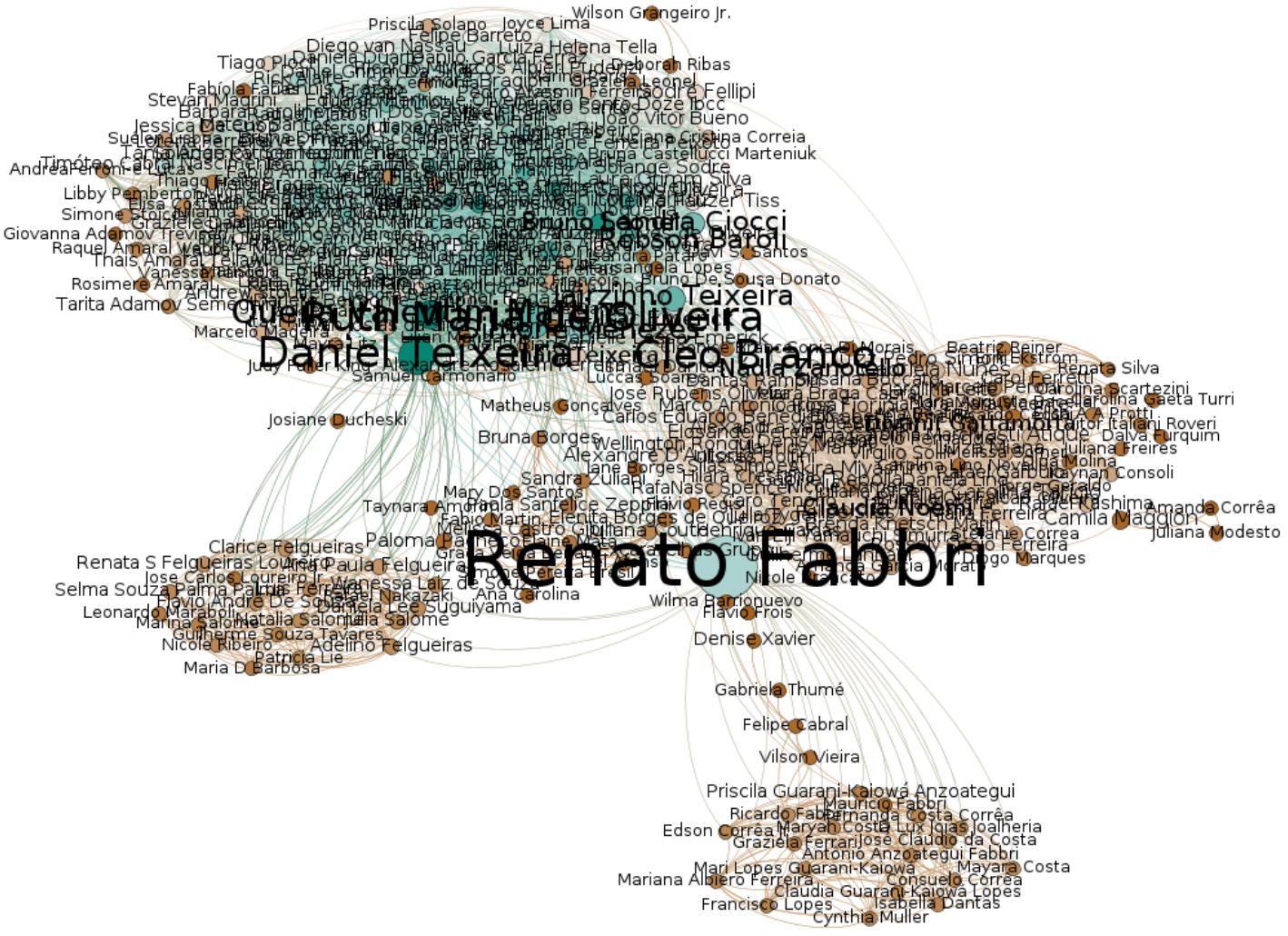
Grafos e PageRank

PageRank



Redes Sociais

Rede de Amizades do Facebook



Redes Sociais

Rede de Postagens e Curtidas

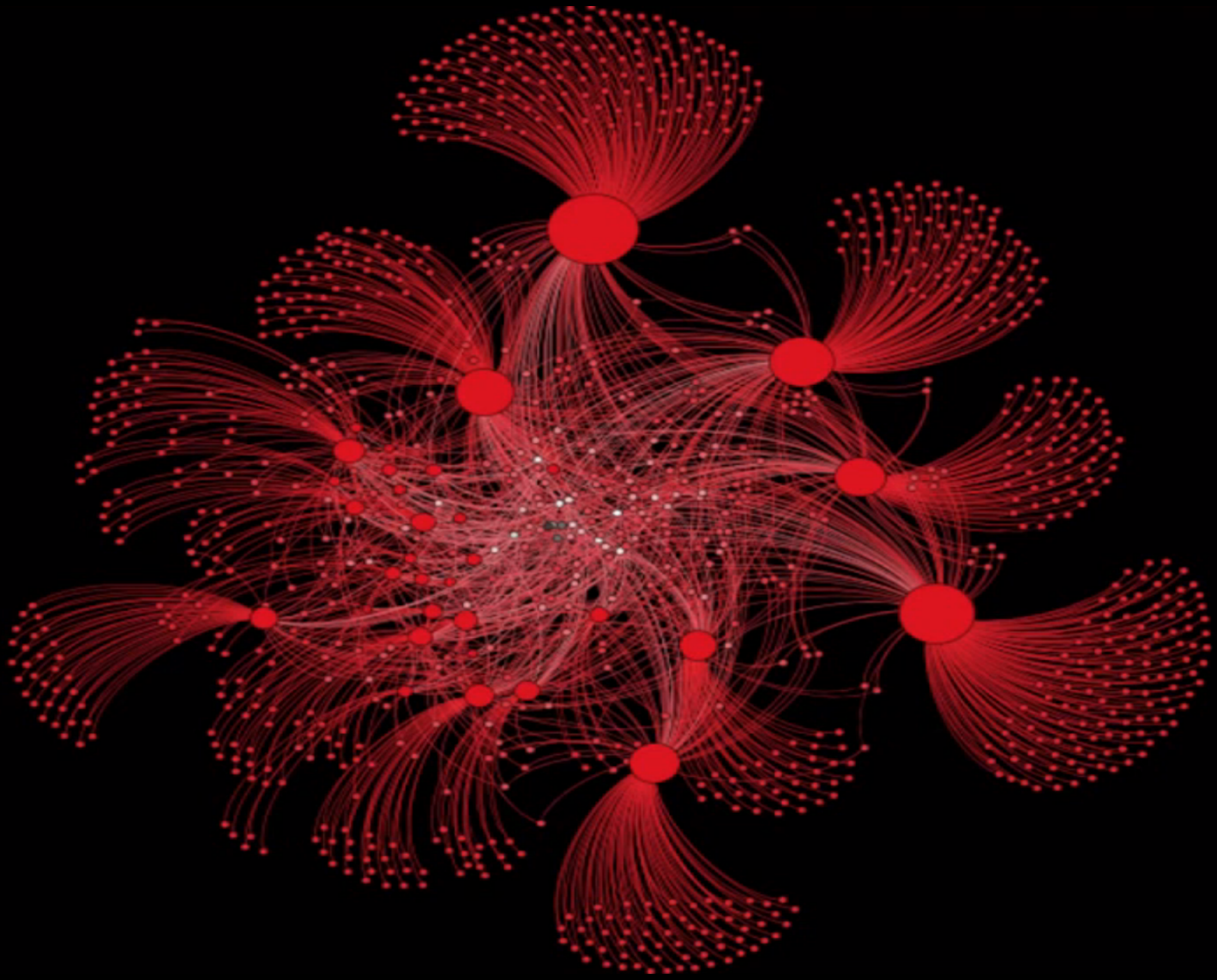


FIGURE 3

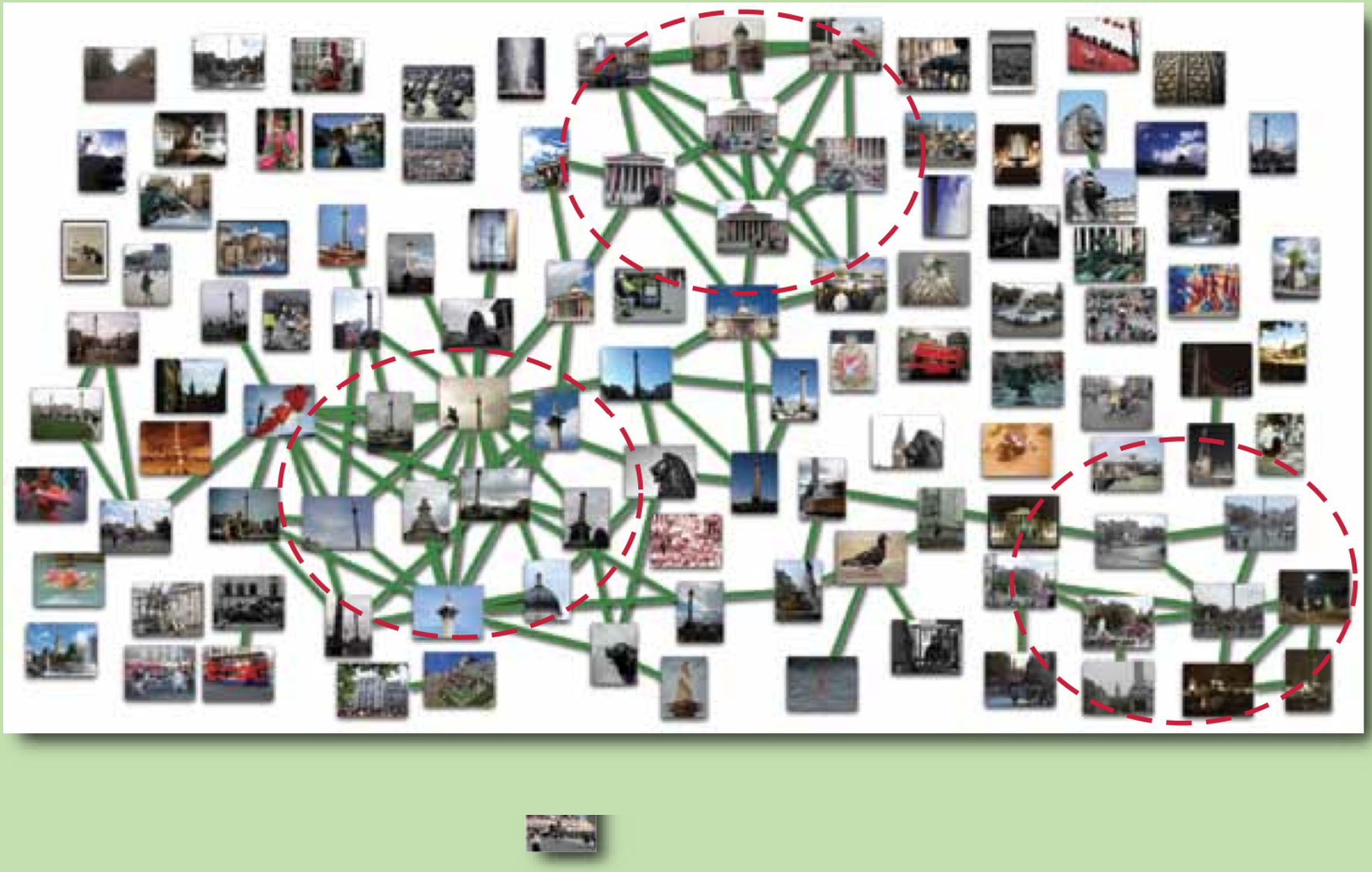
A 3D Model of a Scene Is Generated from Photographs

Crandall & Snavely ACM Queue 2012

FIGURE 6

Grafo de Similaridades Locais

Photo Network for Finding Representative Images



From: Noah Snavely et al.

FIGURE 5

Como as Arestas sao Formadas

SIFT Feature Matching

(a) input photograph

(b) input photo with extracted SIFT features superimposed

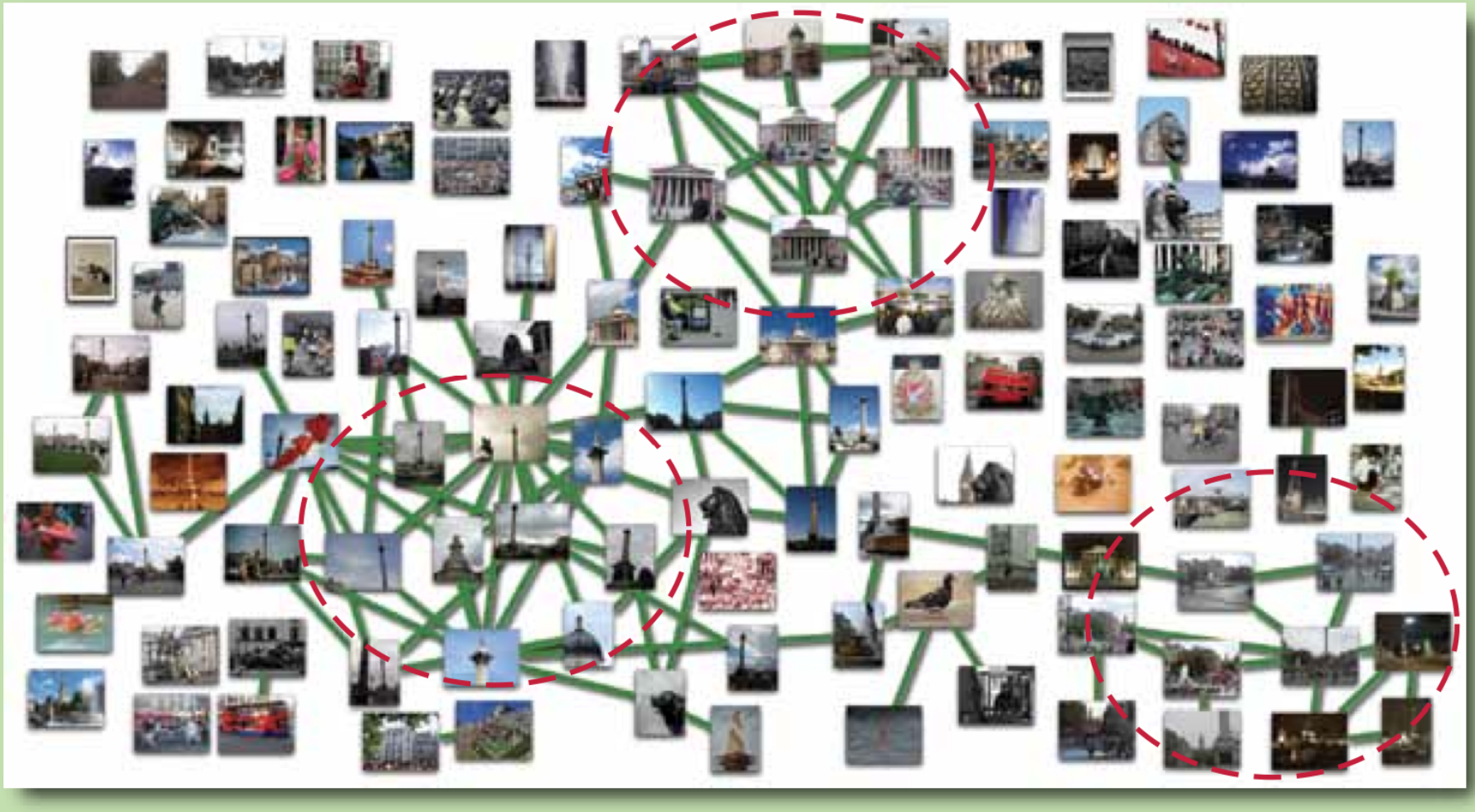
(c) a pair of overlapping images shown side-by-side with matching features connected by black lines

From: Noah Snavely et al.

FIGURE 6

Difusao Global Nessa Rede..?

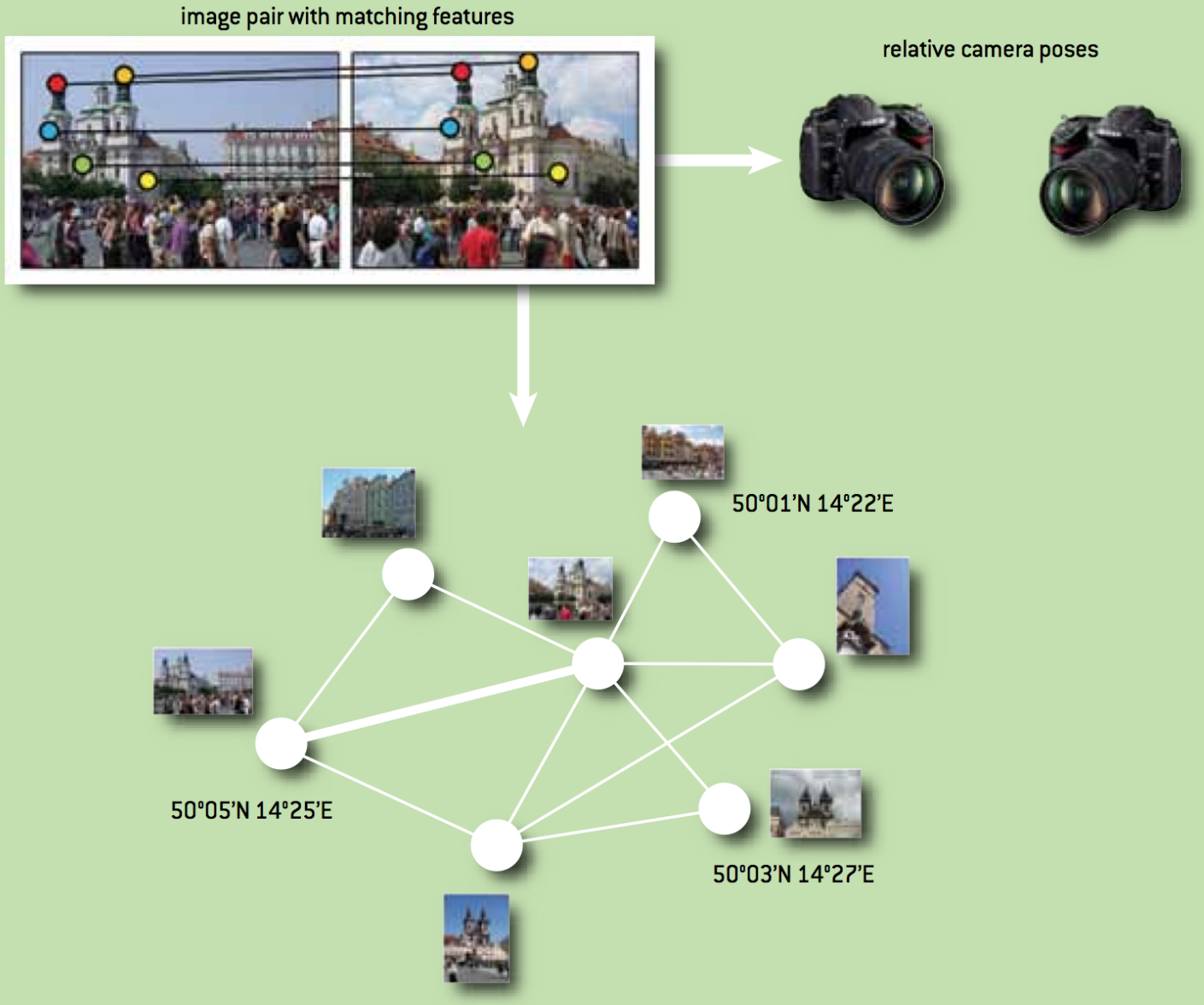
Photo Network for Finding Representative Images



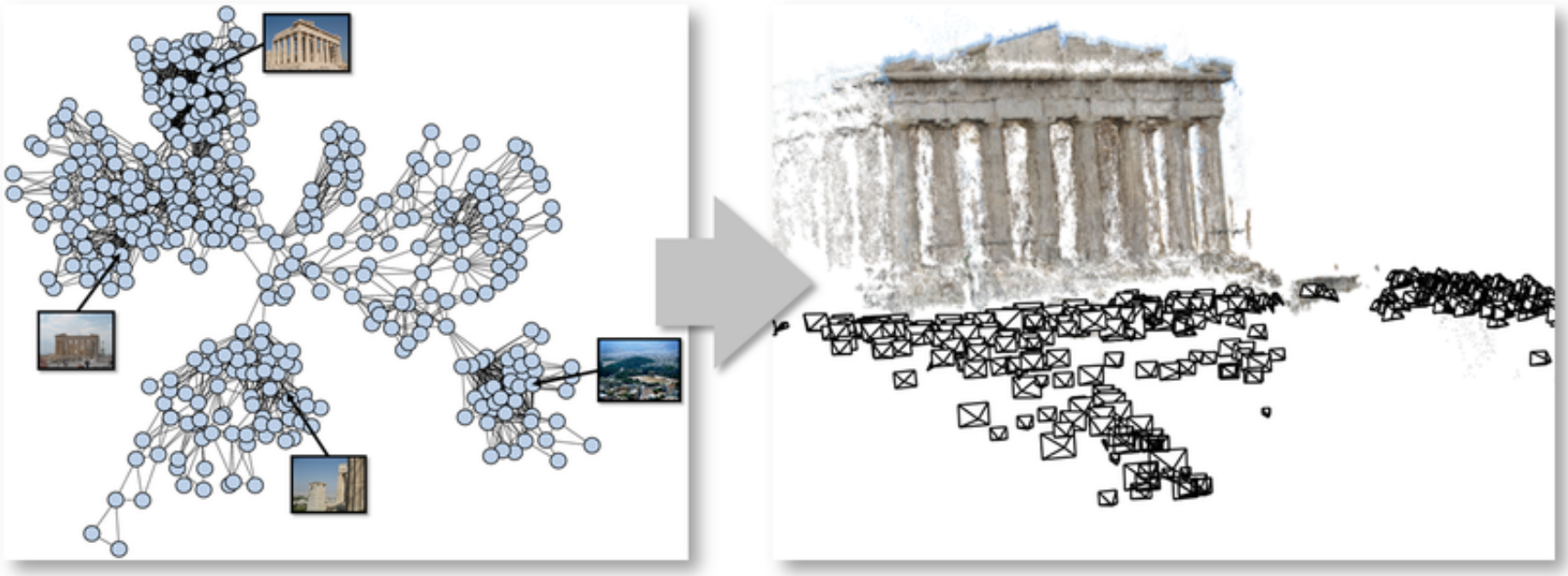
From: Noah Snavely et al.

FIGURE 8

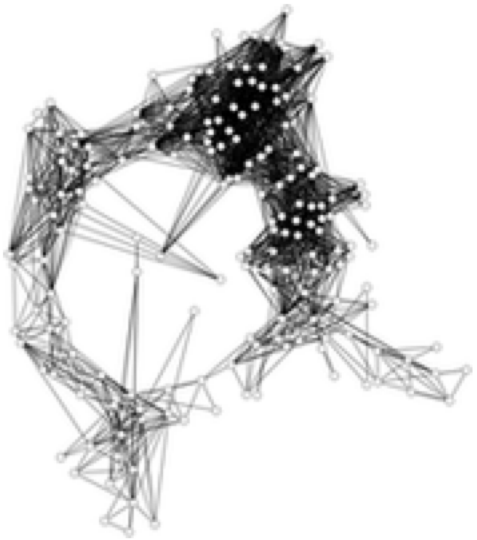
Pose Network



From: Noah Snavely et al.



From: Noah Snavely et al.



An *image graph* for this photo collection.

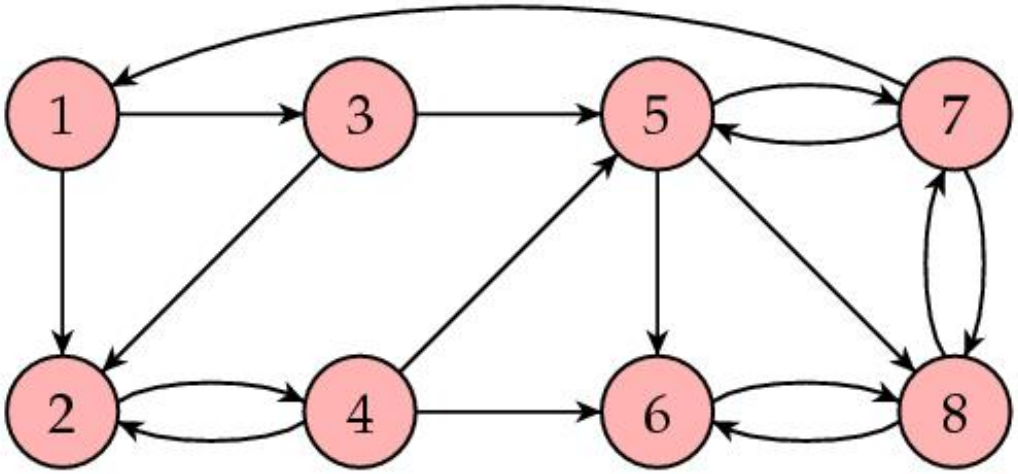
Our computed skeletal graph.

From: Noah Snavely et al.

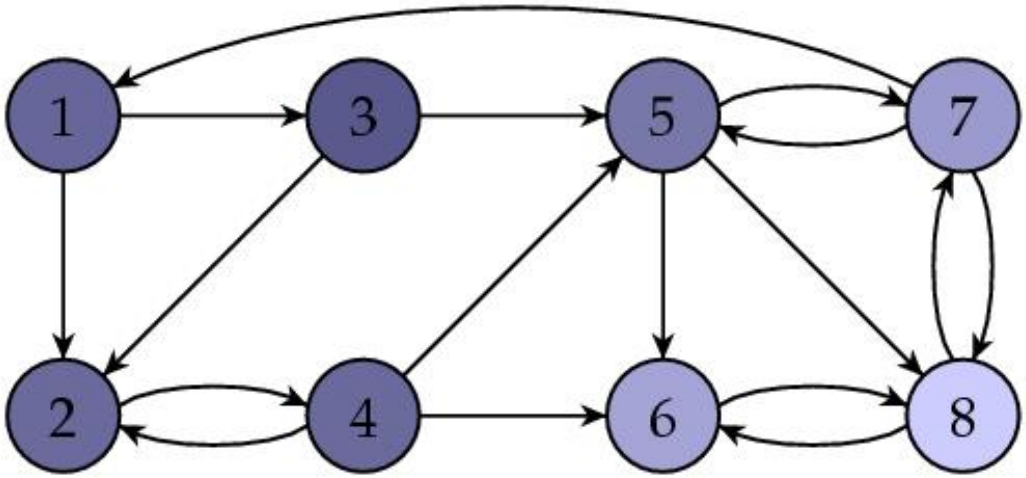
Outros Exemplos de Redes Grandes

- Web: centenas de bilhões de páginas
- Redes de distribuição: energia elétrica, petróleo
- Redes de transportes: rodovias, ferrovias
- Redes de reações químicas
- Redes de fotos cobrindo um determinado lugar
- Redes de similaridade
- Máquinas de estado / Autômatos

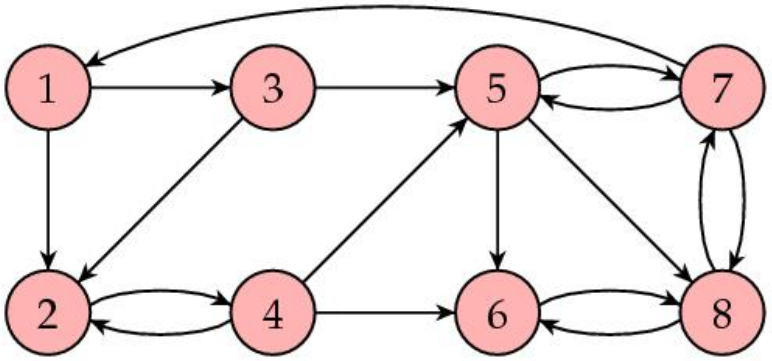
PageRank



PageRank



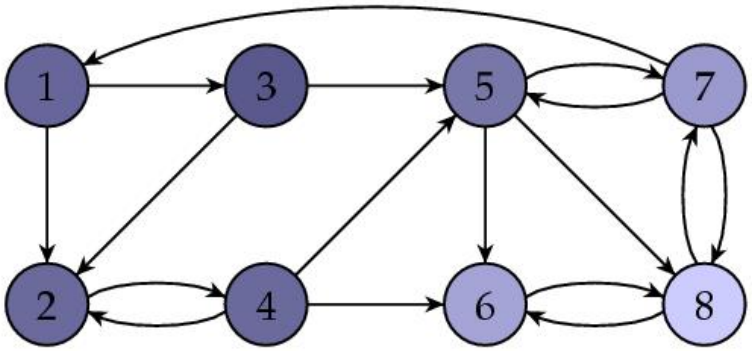
PageRank (Simplificado)



$$\mathbf{H} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1/3 & 0 \\ 1/2 & 0 & 1/2 & 1/3 & 0 & 0 & 0 & 0 \\ 1/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 1/3 & 0 & 0 & 1/3 & 0 \\ 0 & 0 & 0 & 1/3 & 1/3 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 1/3 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 1/3 & 1 & 1/3 & 0 \end{bmatrix}$$

$$I = \begin{bmatrix} 0.0600 \\ 0.0675 \\ 0.0300 \\ 0.0675 \\ 0.0975 \\ 0.2025 \\ 0.1800 \\ 0.2950 \end{bmatrix}$$

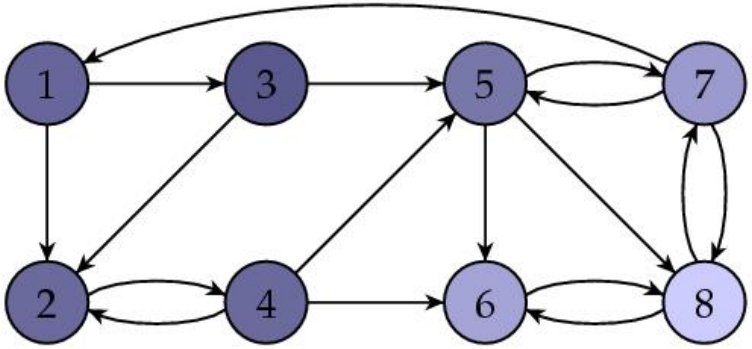
PageRank



$$\mathbf{H} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1/3 & 0 \\ 1/2 & 0 & 1/2 & 1/3 & 0 & 0 & 0 & 0 \\ 1/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 1/3 & 0 & 0 & 1/3 & 0 \\ 0 & 0 & 0 & 1/3 & 1/3 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 1/3 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 1/3 & 1 & 1/3 & 0 \end{bmatrix}$$

$$I = \begin{bmatrix} 0.0600 \\ 0.0675 \\ 0.0300 \\ 0.0675 \\ 0.0975 \\ 0.2025 \\ 0.1800 \\ 0.2950 \end{bmatrix}$$

PageRank



I_0	I_1	I_2	I_3	I_4	...	I_{60}	I_{61}
1	0	0	0	0.0278	...	0.06	0.06
0	0.5	0.25	0.1667	0.0833	...	0.0675	0.0675
0	0.5	0	0	0	...	0.03	0.03
0	0	0.5	0.25	0.1667	...	0.0675	0.0675
0	0	0.25	0.1667	0.1111	...	0.0975	0.0975
0	0	0	0.25	0.1806	...	0.2025	0.2025
0	0	0	0.0833	0.0972	...	0.18	0.18
0	0	0	0.0833	0.3333	...	0.295	0.295

$$I = \begin{bmatrix} 0.0600 \\ 0.0675 \\ 0.0300 \\ 0.0675 \\ 0.0975 \\ 0.2025 \\ 0.1800 \\ 0.2950 \end{bmatrix}$$

PageRank

I_0	I_1	I_2	$I_3 = I$
1	0	0	0
0	1	0	0

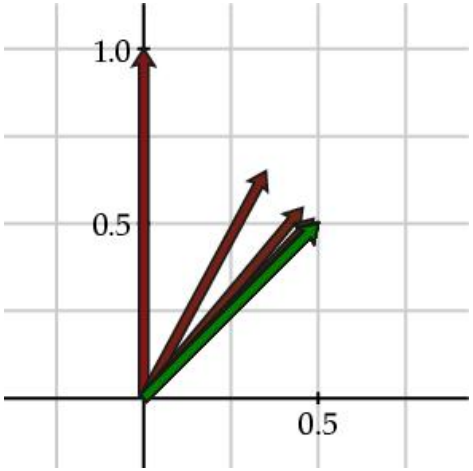
PageRank

- Nós terminais são munidos de arcos virtuais a todos os demais nós

$$\mathrm{S} = \begin{bmatrix} 0 & 0.5 \\ 1 & 0.5 \end{bmatrix}, \qquad \mathrm{I} = \begin{bmatrix} 1/3 \\ 2/3 \end{bmatrix}$$

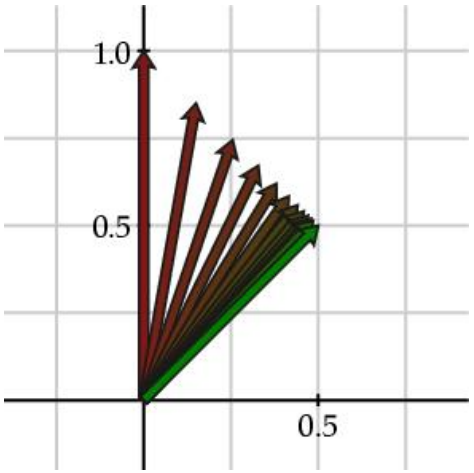
Velocidade de Convergência

$$S = \begin{bmatrix} 0.65 & 0.35 \\ 0.35 & 0.65 \end{bmatrix}$$



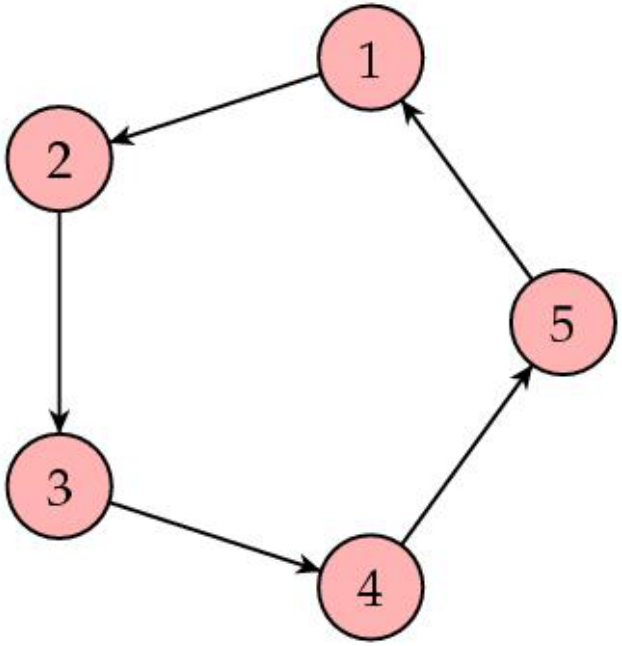
$$\lambda_1 = 1, \quad \lambda_2 = 0.3$$

$$S = \begin{bmatrix} 0.85 & 0.15 \\ 0.15 & 0.85 \end{bmatrix}$$



$$\lambda_1 = 1, \quad \lambda_2 = 0.7$$

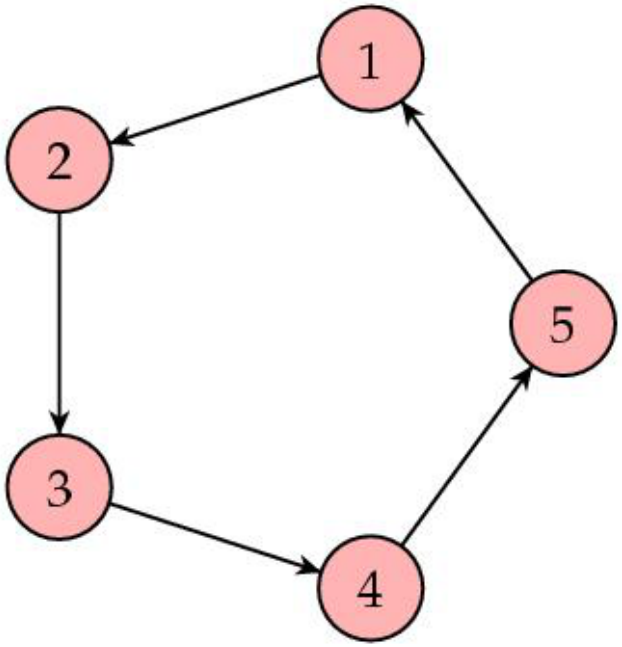
PageRank



0	0	0	0	1
1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0

I_0	I_1	I_2	I_3	I_4	I_5
1	0	0	0	0	1
0	1	0	0	0	0
0	0	1	0	0	0
0	0	0	1	0	0
0	0	0	0	1	0

PageRank



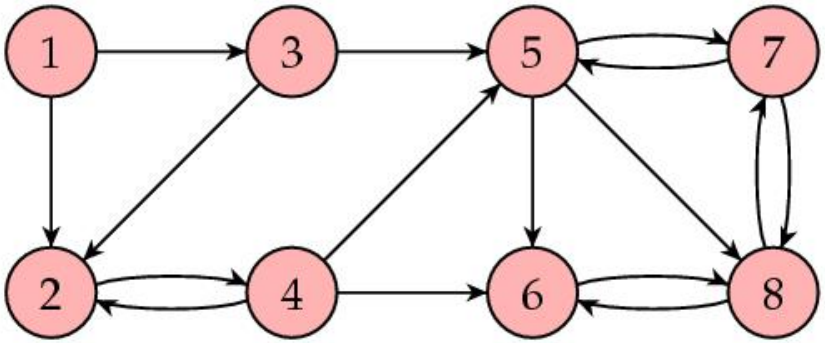
0	0	0	0	1
1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0

I_0	I_1	I_2	I_3	I_4	I_5
1	0	0	0	0	1
0	1	0	0	0	0
0	0	1	0	0	0
0	0	0	1	0	0
0	0	0	0	1	0

$$|\lambda_1| = |\lambda_2| = 1$$

Queremos: primitividade S^m

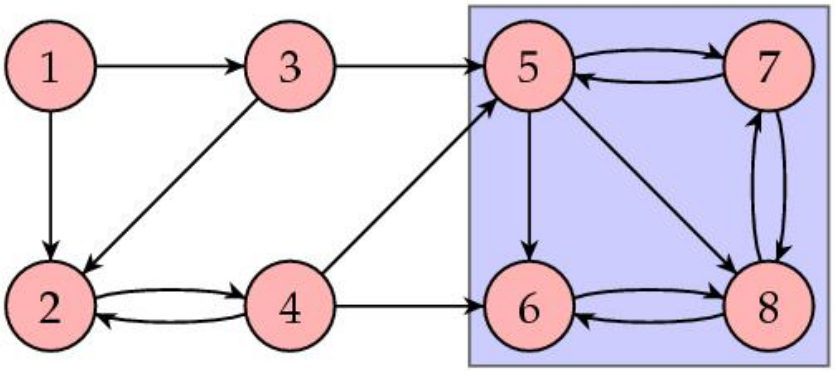
PageRank



$$\mathbf{S} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 1/3 & 0 & 0 & 0 & 0 \\ 1/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 1/3 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1/3 & 1/3 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 1/3 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 1/3 & 1 & 1/2 & 0 \end{bmatrix} \quad I = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0.12 \\ 0.24 \\ 0.24 \\ 0.4 \end{bmatrix}$$

PageRank

- Queremos: Redutível



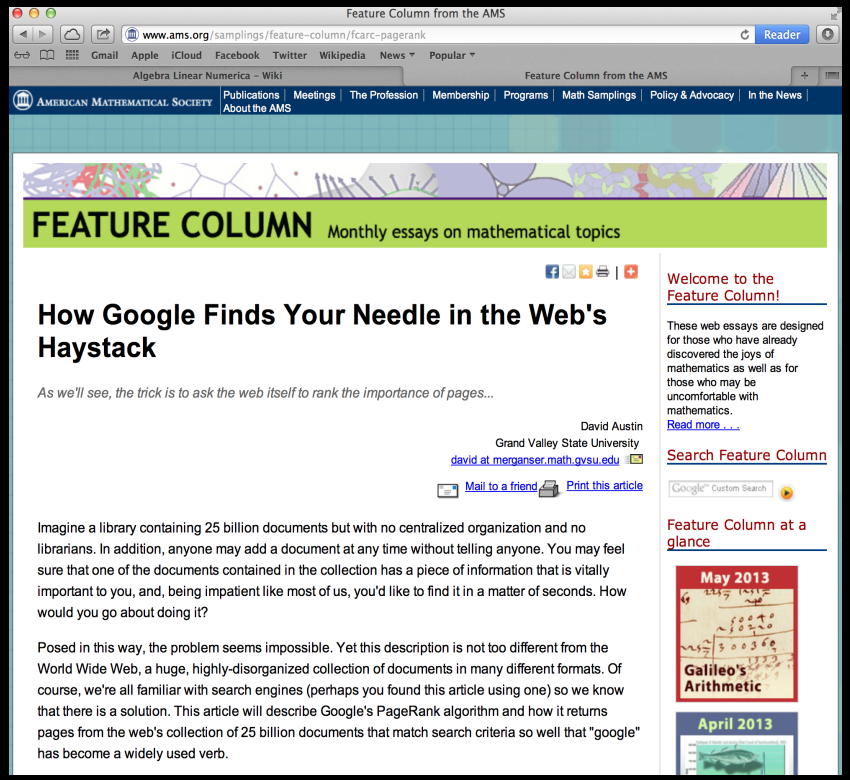
$$\mathbf{S} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 1/3 & 0 & 0 & 0 & 0 \\ 1/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 1/3 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1/3 & 1/3 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 1/3 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 1/3 & 1 & 1/2 & 0 \end{bmatrix} \quad I = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0.12 \\ 0.24 \\ 0.24 \\ 0.4 \end{bmatrix}$$

Outros Aplicações de Idéias Similares

- Aprendizado de Máquina
- Fazer um Robô aprender a andar automaticamente
- Reconhecimento de Faces sob variações de pose
  - Grafo de similaridades locais
  - Processo de Markov integra distâncias globais
  - Autovetores permitem direcionar análise, filtrar, etc.

Fim de Aula

Bibliografia



- Papers Originais de Page & Brin
- http://www.ams.org/samplings/feature-column/fcarc-pagerank
- http://labmacambira.sourceforge.net/redes
- http://www.youtube.com/watch?v=PHdnddIsjGM
- Noah Snavely's website for 3D computer vision stuff
- Restante: Ricardo Fabbri, Ph.D.
- (c) creative commons share-alike