

Álgebra Linear Numérica

PageRank: Uma Aplicação de Álgebra Linear

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Faça muitas perguntas
Somos todos humanos

Plano de Aula

Vimos

- Matrizes e Grafos
- Matriz de Adjacência tem importância profunda
- Normalizando matrizes de adjacência W :
 - Matrizes estocásticas P
 - Grafo vira cadeia de Markov
- Multiplicação de matrizes permite difusão no grafo

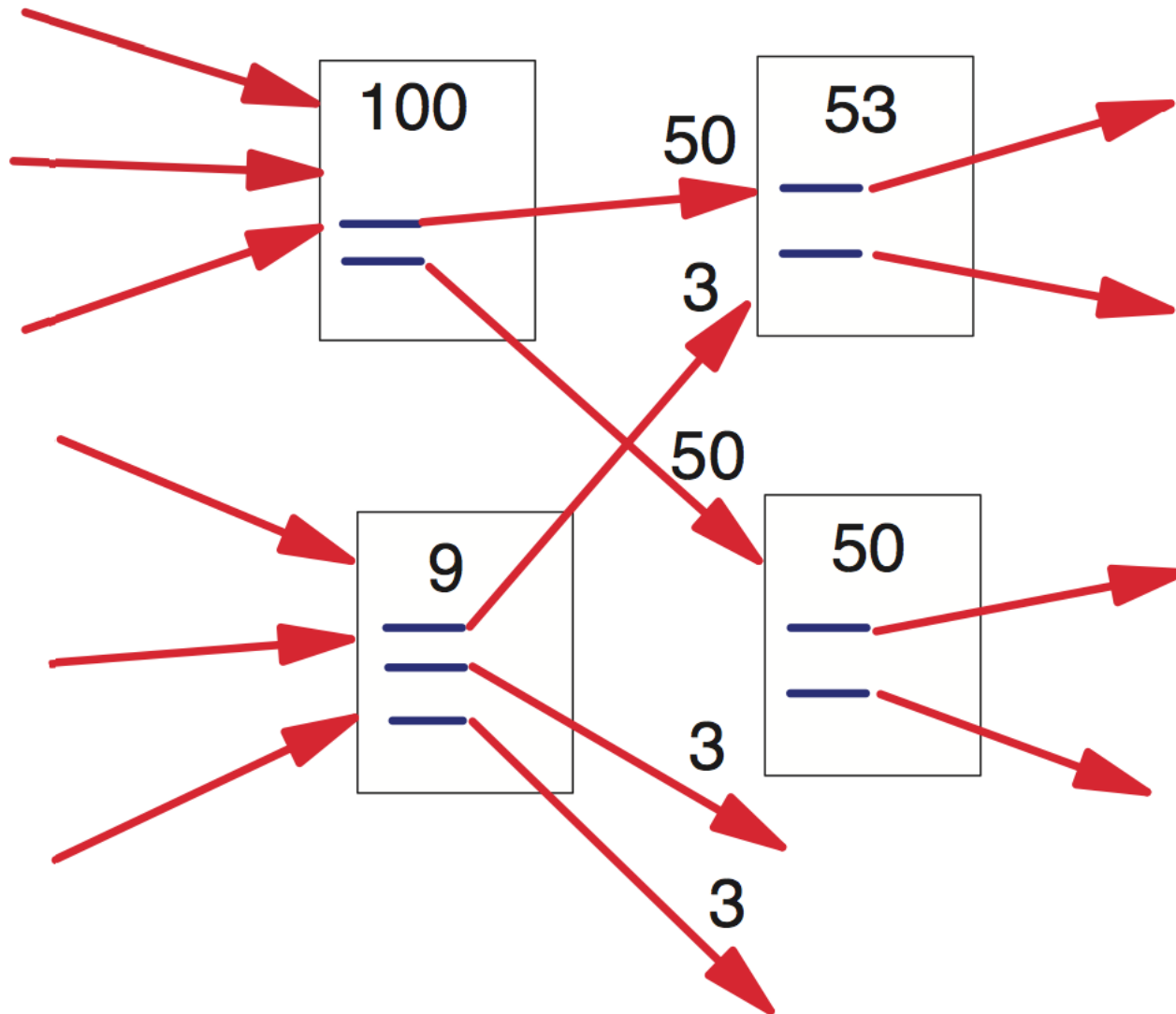
Plano de Aula

Hoje, 3/Julho/2013

- Grafos e o conceito de PageRank
- Grafos como Cadeias de Markov
- Matrizes de Adjacência Estocásticas
- PageRank como limite de um processo de Markov
- PageRank como autovetor da matriz de Markov
- Método das potências para autovetores

Grafos e PageRank

PageRank



Redes Sociais



Redes Sociais

Rede de Postagens e Curtidas

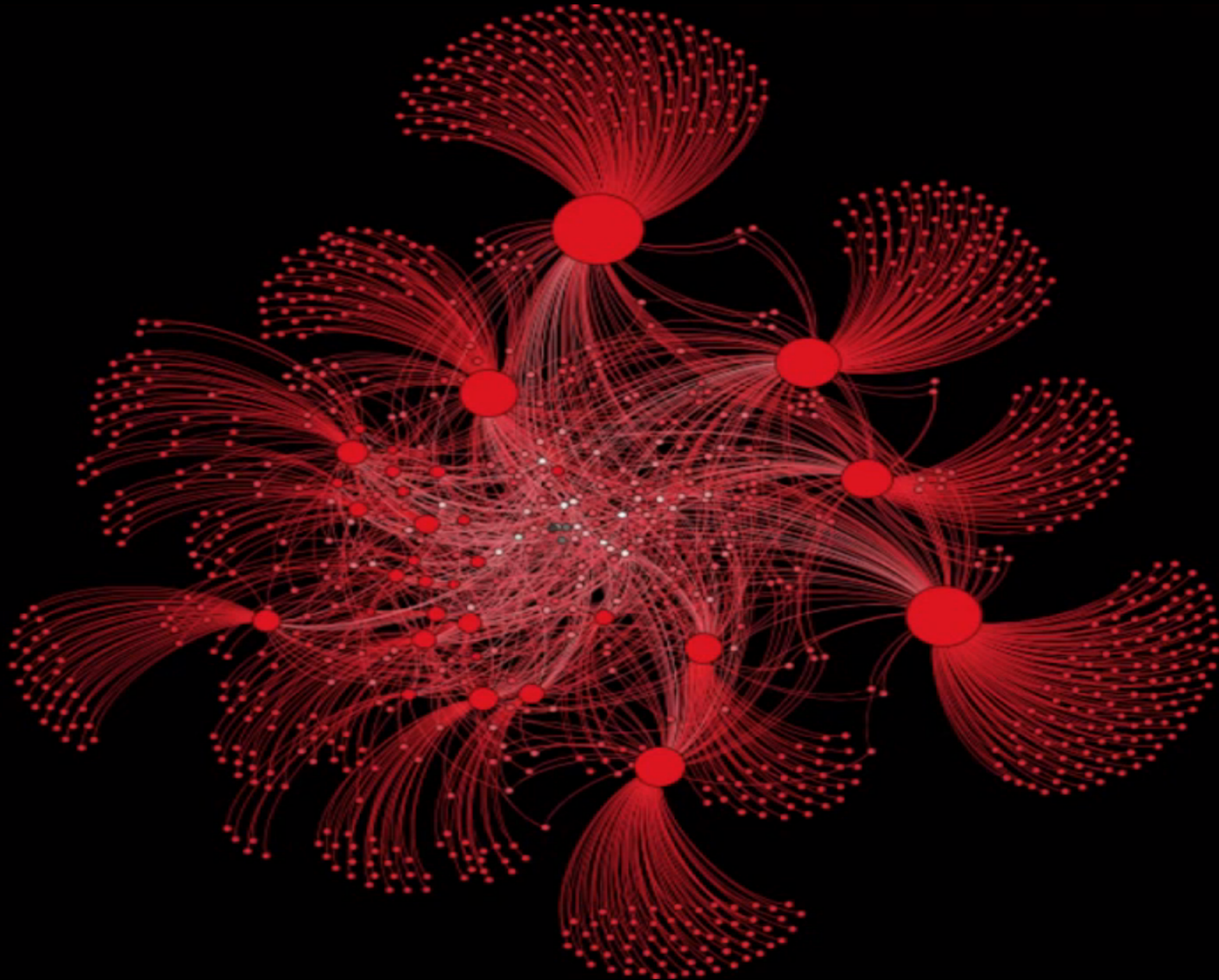


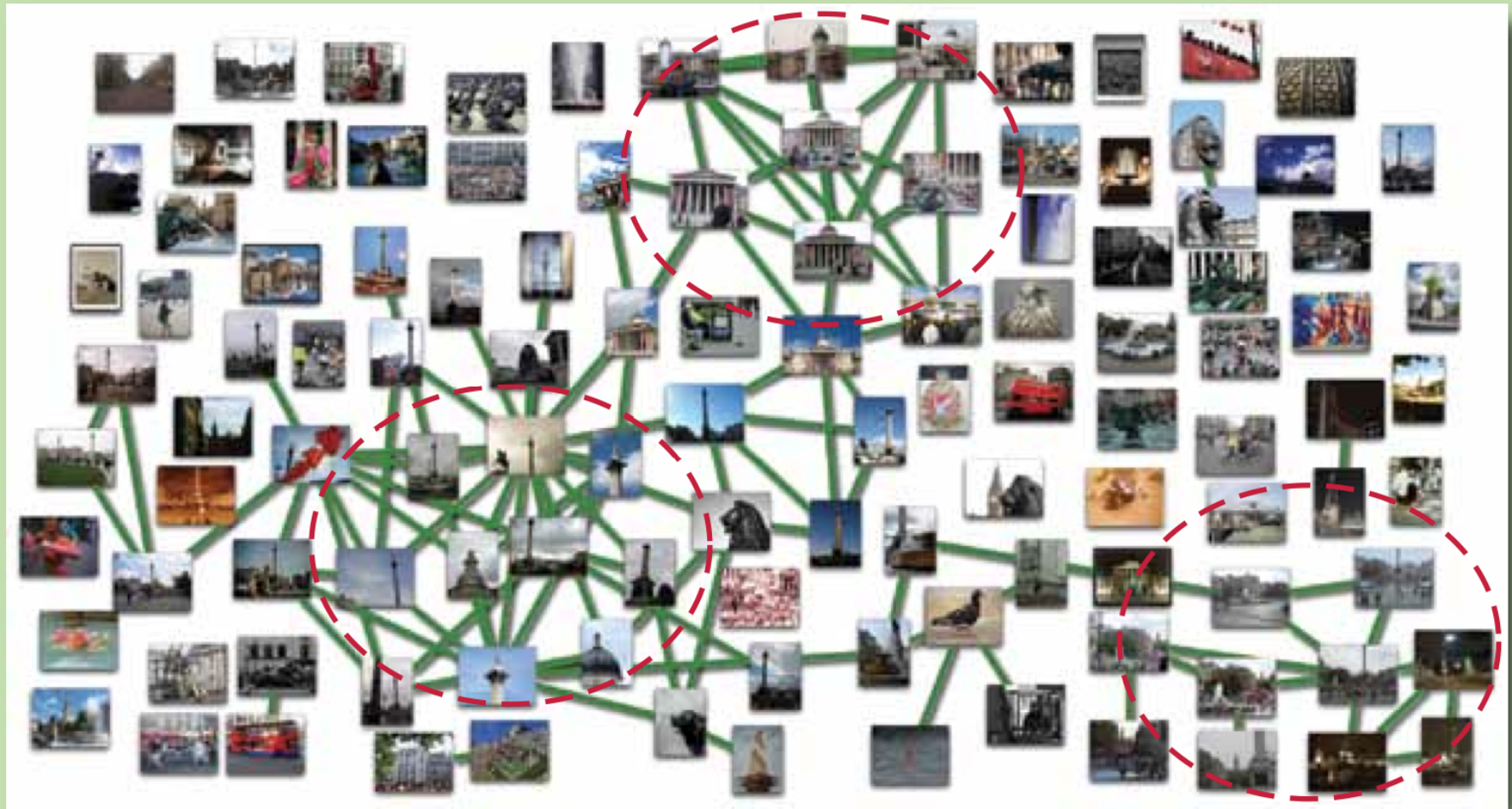
FIGURE 3

A 3D Model of a Scene Is Generated from Photographs



Grafo de Similaridades Locais

Photo Network for Finding Representative Images



Como as Arestas sao Formadas

SIFT Feature Matching

(a) input photograph



(b) input photo with extracted SIFT features superimposed



(c) a pair of overlapping images shown side-by-side with matching features connected by black lines



Difusao Global Nessa Rede..?

Photo Network for Finding Representative Images

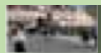
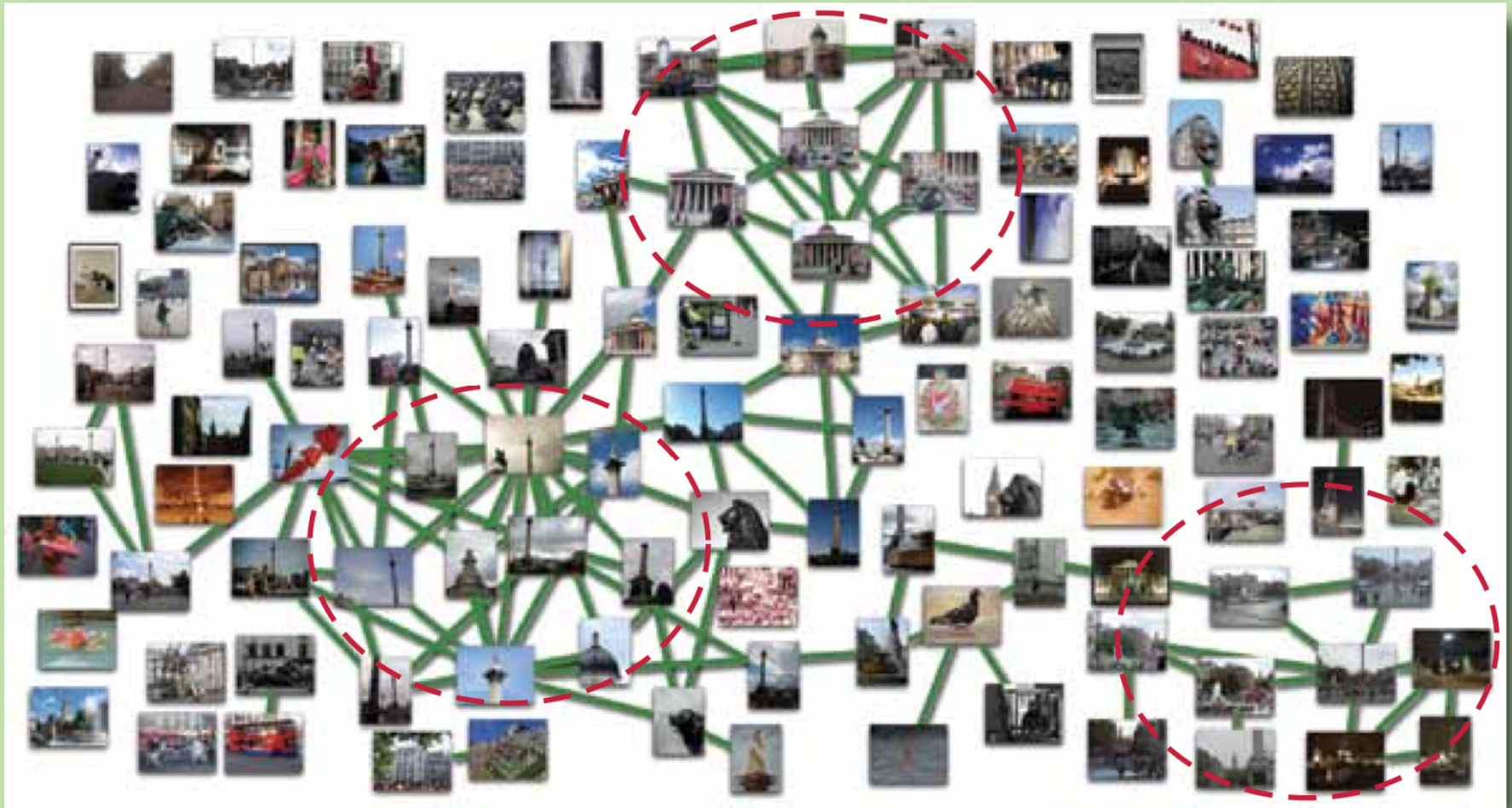
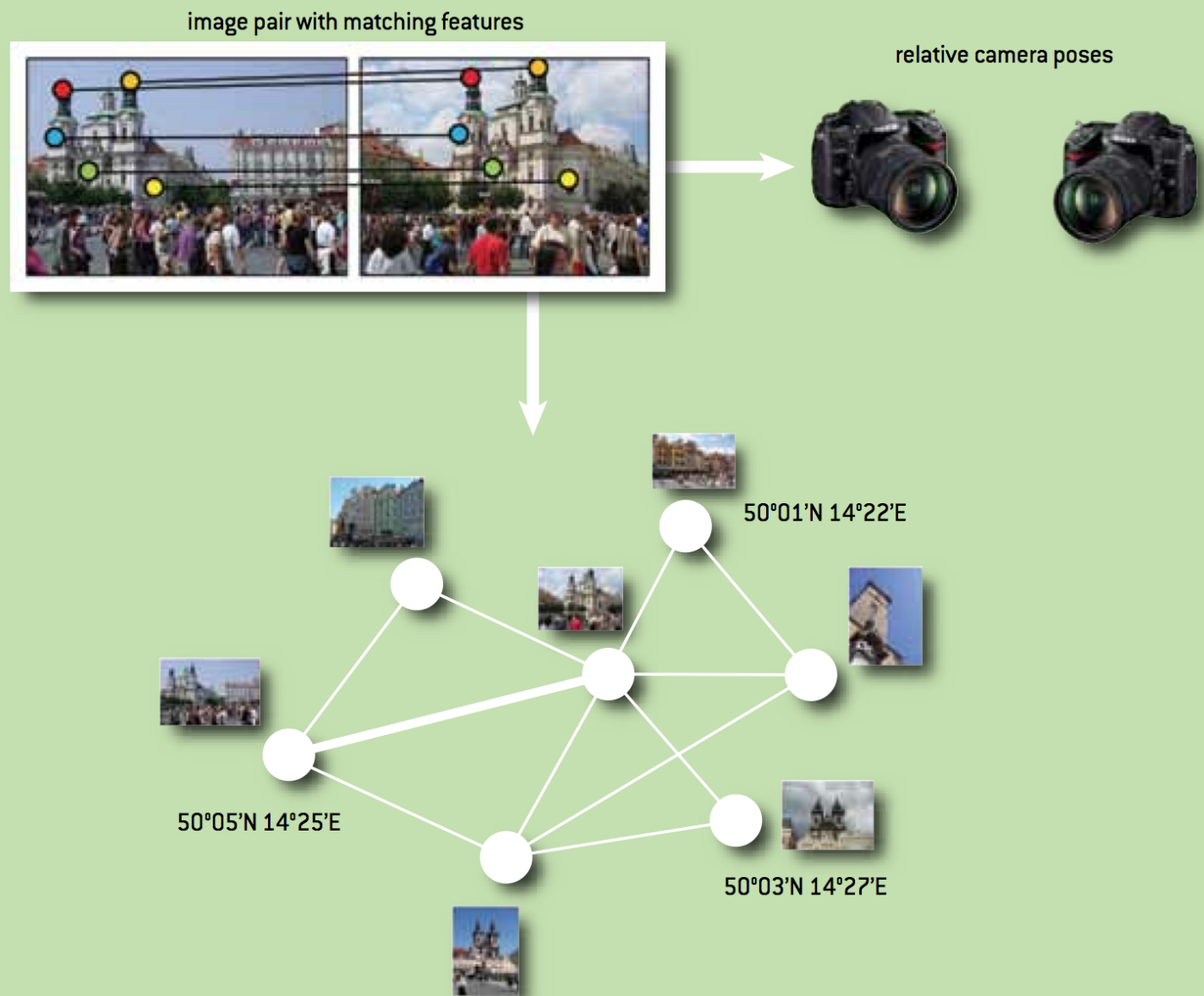
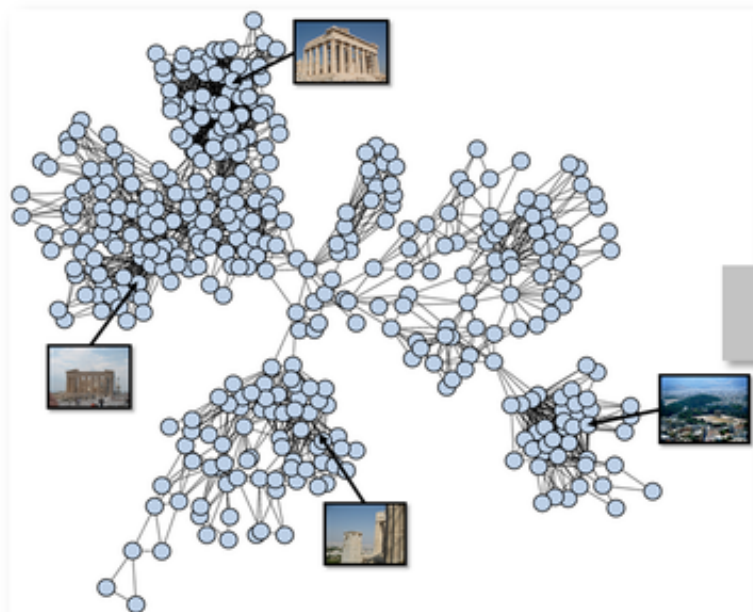
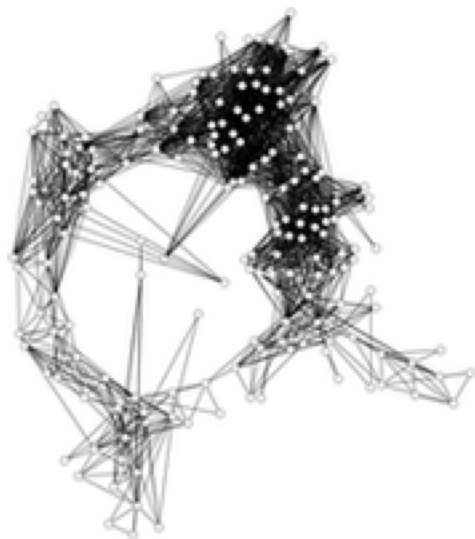


FIGURE 8

Pose Network





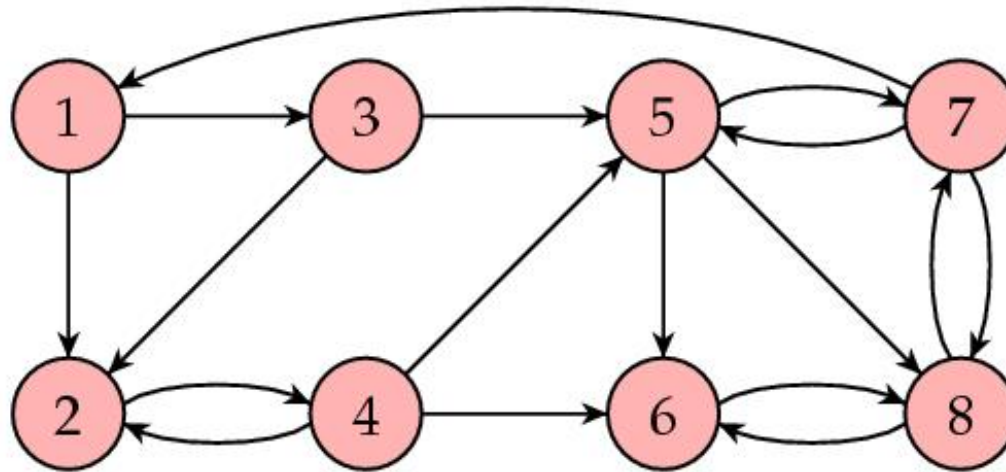


An *image graph* for this photo collection. Our computed skeletal graph.

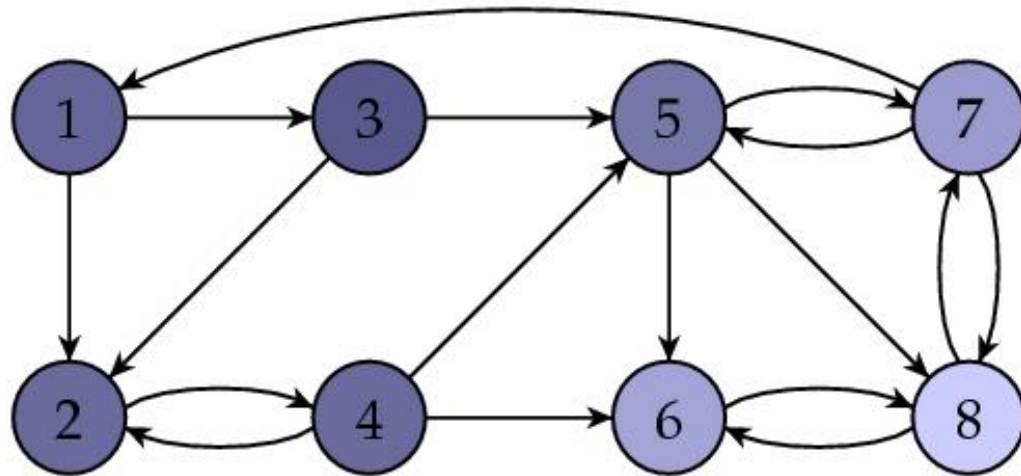
Outros Exemplos de Redes Grandes

- Web: centenas de bilhões de páginas
- Redes de distribuição: energia elétrica, petróleo
- Redes de transportes: rodovias, ferrovias
- Redes de reações químicas
- Redes de fotos cobrindo um determinado lugar
- Redes de similaridade
- Máquinas de estado / Autômatos

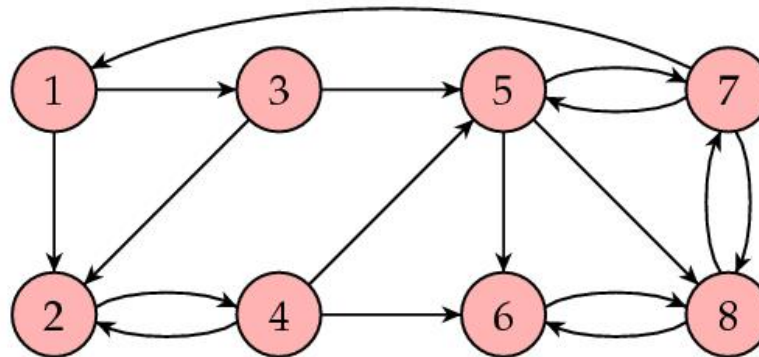
PageRank



PageRank



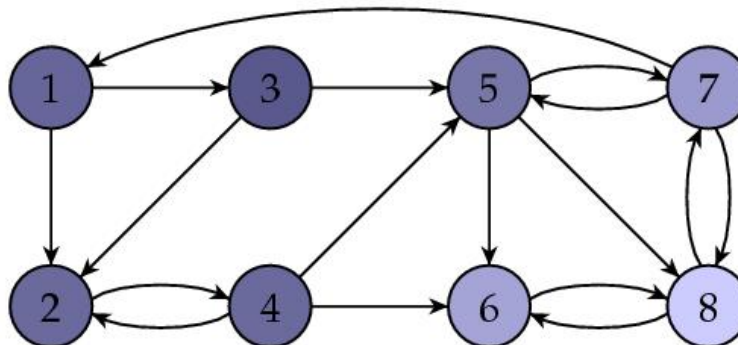
PageRank (Simplificado)



$$H = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1/3 & 0 \\ 1/2 & 0 & 1/2 & 1/3 & 0 & 0 & 0 & 0 \\ 1/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 1/3 & 0 & 0 & 1/3 & 0 \\ 0 & 0 & 0 & 1/3 & 1/3 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 1/3 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 1/3 & 1 & 1/3 & 0 \end{bmatrix}$$

$$I = \begin{bmatrix} 0.0600 \\ 0.0675 \\ 0.0300 \\ 0.0675 \\ 0.0975 \\ 0.2025 \\ 0.1800 \\ 0.2950 \end{bmatrix}$$

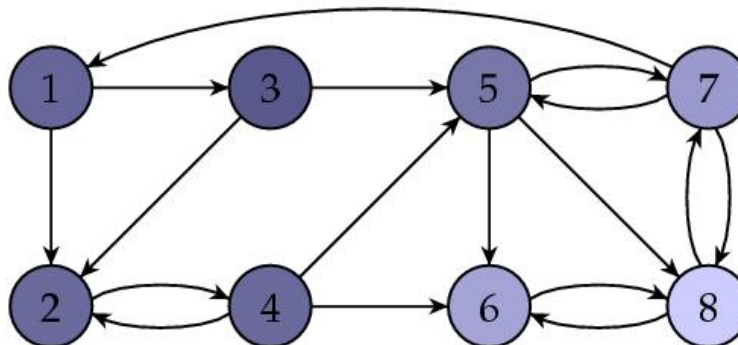
PageRank



$$H = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1/3 & 0 \\ 1/2 & 0 & 1/2 & 1/3 & 0 & 0 & 0 & 0 \\ 1/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 1/3 & 0 & 0 & 1/3 & 0 \\ 0 & 0 & 0 & 1/3 & 1/3 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 1/3 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 1/3 & 1 & 1/3 & 0 \end{bmatrix}$$

$$I = \begin{bmatrix} 0.0600 \\ 0.0675 \\ 0.0300 \\ 0.0675 \\ 0.0975 \\ 0.2025 \\ 0.1800 \\ 0.2950 \end{bmatrix}$$

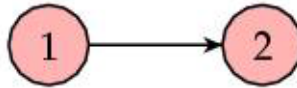
PageRank



I_0	I_1	I_2	I_3	I_4	...	I_{60}	I_{61}
1	0	0	0	0.0278	...	0.06	0.06
0	0.5	0.25	0.1667	0.0833	...	0.0675	0.0675
0	0.5	0	0	0	...	0.03	0.03
0	0	0.5	0.25	0.1667	...	0.0675	0.0675
0	0	0.25	0.1667	0.1111	...	0.0975	0.0975
0	0	0	0.25	0.1806	...	0.2025	0.2025
0	0	0	0.0833	0.0972	...	0.18	0.18
0	0	0	0.0833	0.3333	...	0.295	0.295

$$I = \begin{bmatrix} 0.0600 \\ 0.0675 \\ 0.0300 \\ 0.0675 \\ 0.0975 \\ 0.2025 \\ 0.1800 \\ 0.2950 \end{bmatrix}$$

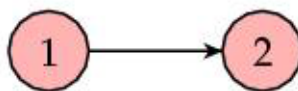
PageRank



I_0	I_1	I_2	$I_3=I$
1	0	0	0
0	1	0	0

PageRank

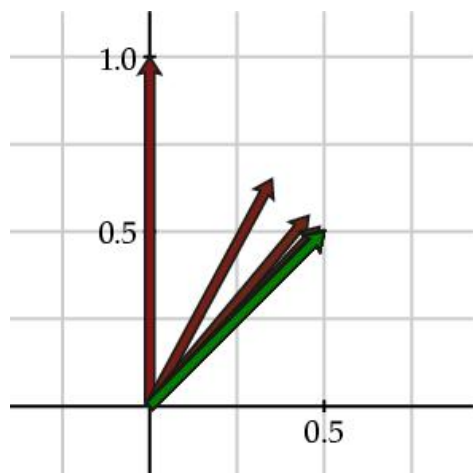
- Nós terminais são munidos de arcos virtuais a todos os demais nós



$$S = \begin{bmatrix} 0 & 0.5 \\ 1 & 0.5 \end{bmatrix}, \quad I = \begin{bmatrix} 1/3 \\ 2/3 \end{bmatrix}$$

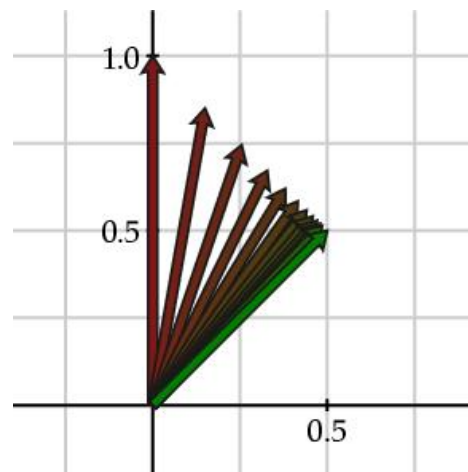
Velocidade de Convergência

$$S = \begin{bmatrix} 0.65 & 0.35 \\ 0.35 & 0.65 \end{bmatrix}$$



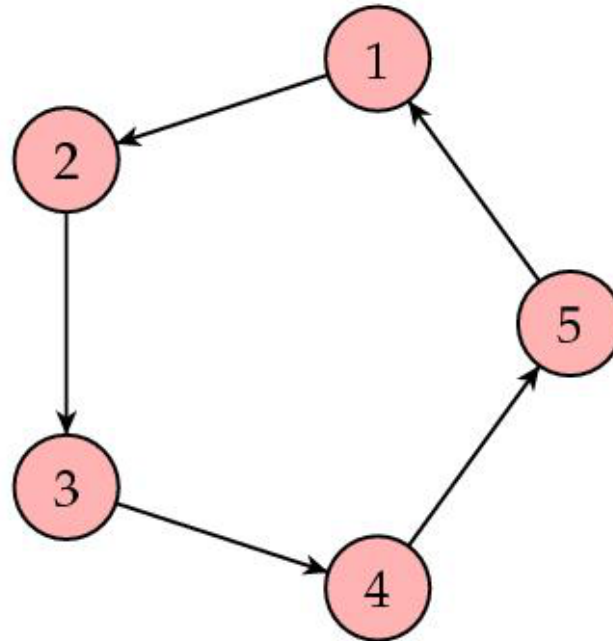
$$\lambda_1 = 1, \quad \lambda_2 = 0.3$$

$$S = \begin{bmatrix} 0.85 & 0.15 \\ 0.15 & 0.85 \end{bmatrix}$$



$$\lambda_1 = 1, \quad \lambda_2 = 0.7$$

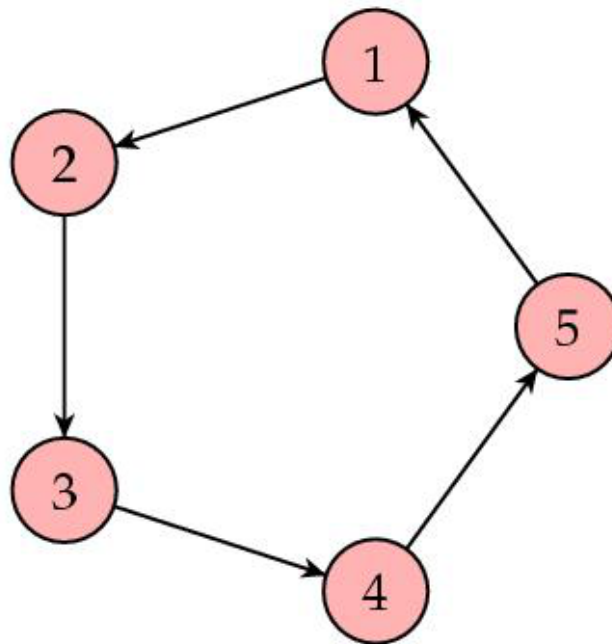
PageRank



0	0	0	0	1
1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0

I_0	I_1	I_2	I_3	I_4	I_5
1	0	0	0	0	1
0	1	0	0	0	0
0	0	1	0	0	0
0	0	0	1	0	0
0	0	0	0	1	0

PageRank



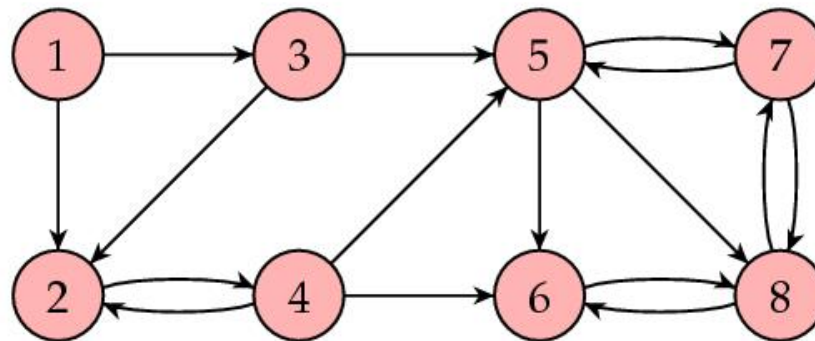
0	0	0	0	1
1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0

l_0	l_1	l_2	l_3	l_4	l_5
1	0	0	0	0	1
0	1	0	0	0	0
0	0	1	0	0	0
0	0	0	1	0	0
0	0	0	0	1	0

$$|\lambda_1| = |\lambda_2| = 1$$

Queremos: primitividade S^m

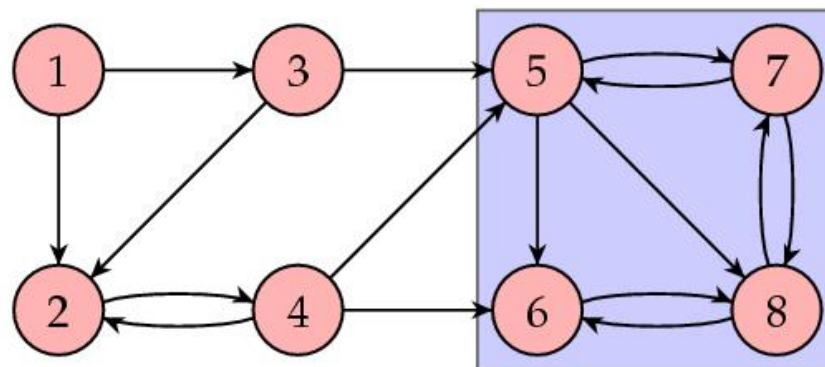
PageRank



$$S = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 1/3 & 0 & 0 & 0 & 0 \\ 1/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 1/3 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1/3 & 1/3 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 1/3 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 1/3 & 1 & 1/2 & 0 \end{bmatrix} \quad I = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0.12 \\ 0.24 \\ 0.24 \\ 0.4 \end{bmatrix}$$

PageRank

- Queremos Redutível



$$S = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 1/3 & 0 & 0 & 0 & 0 \\ 1/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 1/3 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1/3 & 1/3 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 1/3 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 1/3 & 1 & 1/2 & 0 \end{bmatrix} \quad I = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0.12 \\ 0.24 \\ 0.24 \\ 0.4 \end{bmatrix}$$

Outros Aplicações de Idéias Similares

- Aprendizado de Máquina
- Fazer um Robô aprender a andar automaticamente
- Reconhecimento de Faces sob variações de pose
 - Grafo de similaridades locais
 - Processo de Markov integra distâncias globais
 - Autovetores permitem direcionar análise, filtrar, etc.

Fim de Aula

Bibliografia



- Papers Originais de Page & Brin
- <http://www.ams.org/samplings/feature-column/fcarc-pagerank>
- <http://labmacambira.sourceforge.net/redes>
- <http://www.youtube.com/watch?v=PHdnddlsjGM>
- Restante: Ricardo Fabbri, Ph.D.
- (c) creative commons share-alike